A **rational equation** is an equation that contains one or more rational expressions.

To solve a rational function, start by multiplying each term by the least common denominator (LCD).
Example 1: Solving Rational Equations

Solve the equation \( x - \frac{18}{x} = 3 \).

\[
x(x) - \frac{18}{x} (x) = 3(x) \quad \text{Multiply each term by the LCD, } x.
\]
\[
x^2 - 18 = 3x \quad \text{Simplify. Note that } x \neq 0.
\]
\[
x^2 - 3x - 18 = 0 \quad \text{Write in standard form.}
\]
\[
(x - 6)(x + 3) = 0 \quad \text{Factor.}
\]
\[
x - 6 = 0 \text{ or } x + 3 = 0 \quad \text{Apply the Zero Product Property.}
\]
\[
x = 6 \text{ or } x = -3 \quad \text{Solve for } x.
\]
### Example 1 Continued

**Check**

\[ x - \frac{18}{x} = 3 \]

<table>
<thead>
<tr>
<th>( 6 - \frac{18}{6} )</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 - 3</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3 ✔</td>
</tr>
</tbody>
</table>

\[ x - \frac{18}{x} = 3 \]

<table>
<thead>
<tr>
<th>((-3) - \frac{18}{(-3)})</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3 + 6</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3 ✔</td>
</tr>
</tbody>
</table>
Check It Out! Example 1a

Solve the equation \( \frac{10}{3} = \frac{4}{x} + 2 \).

\[
\frac{10}{3} (3x) = \frac{4}{x} (3x) + 2(3x)
\]

Multiply each term by the LCD, 3x.

\[
10x = 12 + 6x
\]

Simplify. Note that \( x \neq 0 \).

\[
4x = 12
\]

Combine like terms.

\[
x = 3
\]

Solve for \( x \).
8-5 Solving Rational Equations and Inequalities

If you get a solution to a rational equation that matches where the rational function is undefined, then that value is an **extraneons solution**.

\[
\frac{5x}{x - 2} = \frac{3x + 4}{x - 2}
\]

Multiply each term by the LCD, \(x - 2\).

Divide out common factors.

Simplify. Note that \(x \neq 2\).

Solve for \(x\).

The solution \(x = 2\) is extraneons because it makes the denominators of the original equation equal to 0. Therefore, the equation has no solution.
Example 2A Continued

Check Substitute 2 for \( x \) in the original equation.

\[
\frac{5x}{x - 2} = \frac{3x + 4}{x - 2}
\]

\[
\frac{5(2)}{2 - 2} \quad \frac{3(2) + 4}{2 - 2}
\]

\[
\frac{10}{0} \quad \frac{10}{0}
\]

Division by 0 is undefined.
Example 2B: Extraneous Solutions

Solve each equation.

\[
\frac{2x - 5}{x - 8} + \frac{x}{2} = \frac{11}{x - 8}
\]

Multiply each term by the LCD, 2(x – 8).

\[
\frac{2x - 5}{x - 8} \cdot 2(x - 8) + \frac{x}{2} \cdot 2(x - 8) = \frac{11}{x - 8} \cdot 2(x - 8)
\]

Divide out common factors.

\[
\frac{2x - 5}{x - 8} \cdot 2(x - 8) + \frac{x}{2} \cdot 2(x - 8) = \frac{11}{x - 8} \cdot 2(x - 8)
\]

\[
2(2x - 5) + x(x - 8) = 11 \cdot 2 \quad \text{Simplify. Note that } x \neq 8.
\]

\[
4x - 10 + x^2 - 8x = 22 \quad \text{Use the Distributive Property.}
\]
Example 2B Continued

\[ x^2 - 4x - 32 = 0 \]  \hspace{1cm} \text{Write in standard form.}

\[(x - 8)(x + 4) = 0 \]  \hspace{1cm} \text{Factor.}

\[ x - 8 = 0 \text{ or } x + 4 = 0 \]  \hspace{1cm} \text{Apply the Zero Product Property.}

\[ x = 8 \text{ or } x = -4 \]  \hspace{1cm} \text{Solve for } x.

The solution \( x = 8 \) is extraneous because it makes the denominator of the original equation equal to 0. The only solution is \( x = -4 \).
Check It Out! Example 2a

Solve the equation \( \frac{16}{x^2 - 16} = \frac{2}{x - 4} \).

Multiply each term by the LCD, \((x - 4)(x + 4)\).

\[
\frac{16}{(x - 4)(x + 4)} (x - 4)(x + 4) = \frac{2}{x - 4} (x - 4)(x + 4)
\]

Divide out common factors.

\[
\frac{16}{(x - 4)(x + 4)} (x - 4)(x + 4) = \frac{2}{x - 4} (x - 4)(x + 4)
\]

\[16 = 2x + 8\]  \(\text{Simplify. Note that } x \neq \pm 4.\)

\[x = 4\]  \(\text{Solve for } x.\)

The solution \(x = 4\) is extraneous because it makes the denominators of the original equation equal to 0. Therefore, the equation has no solution.
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