Applications of Rational Functions
Algebra 2

1. On the web site http://www.eren.doe.gov/consumerinfo/energy_savers/appliancesbody.html there is a list of yearly cost for electricity for common household appliances.

<table>
<thead>
<tr>
<th>Appliance</th>
<th>Average Cost/year in electricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Computer</td>
<td>$9</td>
</tr>
<tr>
<td>Television</td>
<td>$13</td>
</tr>
<tr>
<td>Microwave</td>
<td>$13</td>
</tr>
<tr>
<td>Dishwasher</td>
<td>$51</td>
</tr>
<tr>
<td>Clothes Dryer</td>
<td>$75</td>
</tr>
<tr>
<td>Washing Machine</td>
<td>$79</td>
</tr>
<tr>
<td>Refrigerator</td>
<td>$92</td>
</tr>
</tbody>
</table>

a. If we assume that a new refrigerator costs $550, determine the total annual cost for a refrigerator that lasts for 15 years. Assume costs include purchase and electricity.
b. Develop a function that gives the annual cost of a refrigerator as a function of the number of years you own the refrigerator.
c. Sketch a graph of that function. What is an appropriate window? Label scale.
d. Since this is a rational function, determine the asymptotes of this function?
e. Explain the meaning of the horizontal asymptote in terms of the refrigerator.
f. If a company offers a refrigerator that costs $1200, but says that it will last at least twenty years, is the refrigerator worth the difference in cost?
2. The function \( C(t) = \frac{5t}{0.01t^2 + 3.3} \) describes the concentration of a drug in the blood stream over time. In this case, the medication was taken orally. \( C \) is measured in micrograms per milliliter and \( t \) is measured in minutes.

   a. Sketch a graph of the function over the first two hours after the dose is given. Label axes.
   b. Determine when the maximum amount of the drug is in the body and the amount at that time.
   c. Explain within the context of the problem the shape of the graph between taking the medication orally \((t = 0)\) and the maximum point. What does the shape of the graph communicate between the maximum point and two hours after taking the drug?
   d. What are the asymptotes of the rational function \( C(t) = \frac{5t}{0.01t^2 + 3.3} \)? What is the meaning of the asymptotes within the context of the problem?
   e. Expand the window of the graph to include negative values for \( t \). Discuss the asymptotes.
Follow-Up Problems
Applications of Rational Functions
Algebra 2

3. Tiffany and Adam are graphing functions at the chalk board. Tiffany is graphing $f(x) = \frac{5x-3}{x}$ and Adam is graphing $g(x) = 5 - \frac{3}{x}$.

   a. The teacher asks both students to first write the domain of each of the functions. What should Tiffany say? What should Adam say?
   
   b) Because these are both rational functions, the teacher asks both students to identify the asymptotes for each of the functions. What should each person say?
   
   c) Sketch the graphs of each function and check on the graphing calculator.
   
   d) Surprised? Write an explanation of why these functions produce the same graph and then point out the easy information that comes from each form.

4. Your family is traveling in your car for 100 miles.
   a) How long will the trip take if you average 30 miles per hour, 55 miles per hour, or 65 miles per hour?
   
   b) Write a function that describes the time it takes to make this trip as a function of your speed.
   
   c) Graph this function. Show asymptotes.
   
   d) What does this graph tell you about the time it will take you to travel depending on the speed of the car?

5. For what positive number is the sum of the number and its reciprocal smallest?
Learning Activity: The following is an actual mathematical model used for Cost-Benefit analysis. The model is a rational function. Read the situation and analyze what the solution should be using the algebraic techniques we have studied.

Application of Rational Functions: Why is it important to understand Rational Functions? Consider the following.

This application is a Cost-Benefit Model. A utility company burns coal to generate electricity. The cost $C$ (in dollars) of removing $p$ amount (percent) of the smokestack pollutants is given by:

$$C = \frac{80,000p}{(100 - p)}$$

Is it possible for the company to remove 100 percent of the pollutants? Discuss why or why not, and support your response by using algebraic analysis on the given model.

What happens if the company does try to remove 100 percent of the pollutants? Will the company be successful at doing so, or will the attempt end in failure, that is, will it be too much expense for the company?

Draw a diagram to show what the consequences of the last question would be. Label the vertical asymptote(s) and analyze their impact on the company’s expense.

You will explain why the denominator of a rational function cannot be zero thus recognizing these values as the places where vertical asymptotes occur (which are disastrous things to have) and graphically what vertical asymptotes look like and mean.